

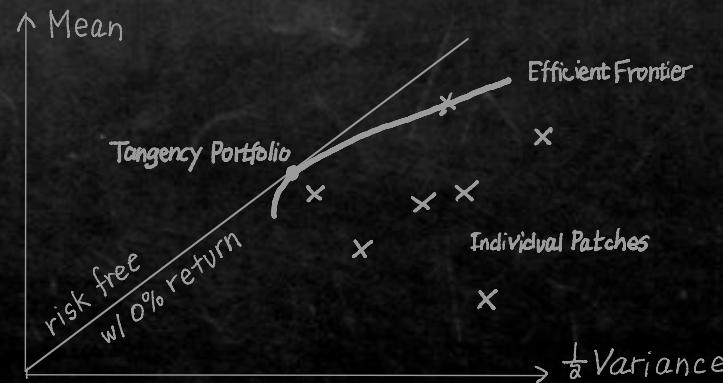
$$dx_i = x_i \sum_{\ell=1}^K p_i^\ell \left\{ f_i^\ell(P^\ell x^\ell) dt + dE_i^\ell \right\}$$

## I. MODELS & COEXISTENCE

## II. MAIN RESULTS: COEVOLUTIONARY STABILITY

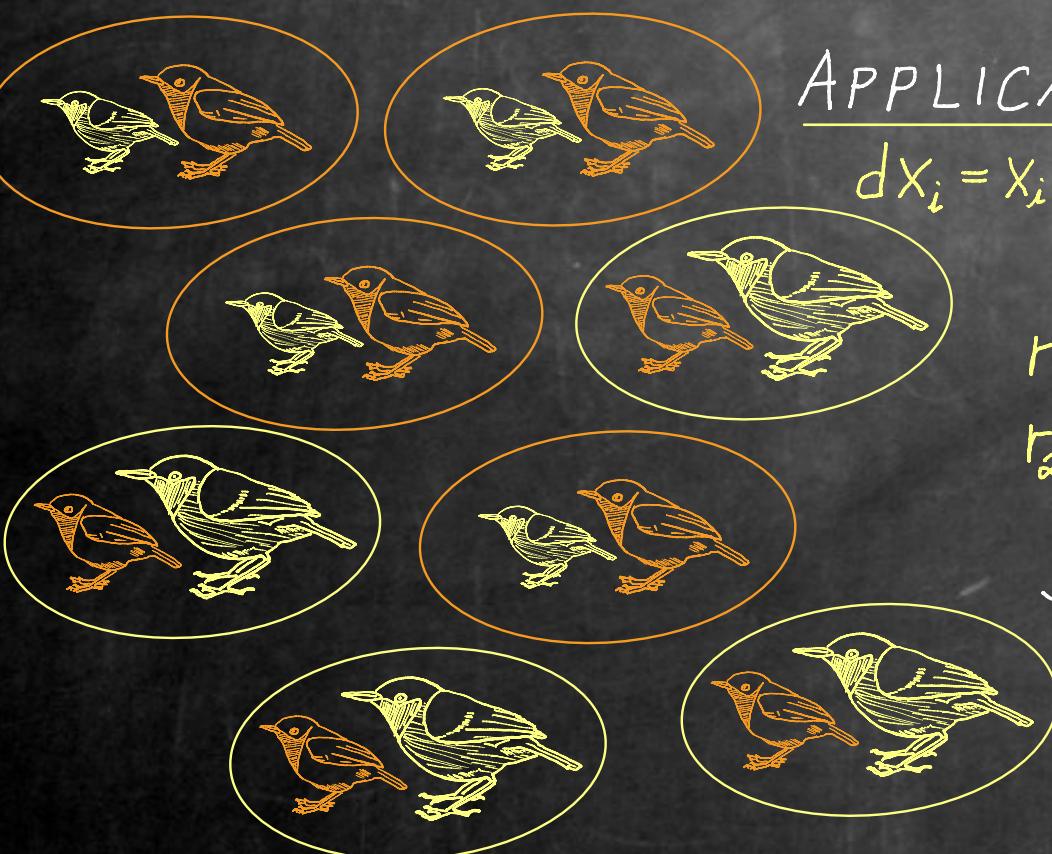
## III. APPLICATIONS: COMPETING SPECIES PREDATOR - PREY

## IV. MODERN PORTFOLIO THEORY & FINALE



APPLICATION: COMPETING SPECIES

$$dx_i = x_i \sum_{j \neq i} P_j^L [(r_j^L - P_1^L x_1 - P_2^L x_2) dt + \sqrt{v} dB_j^L] \quad i=1,2$$



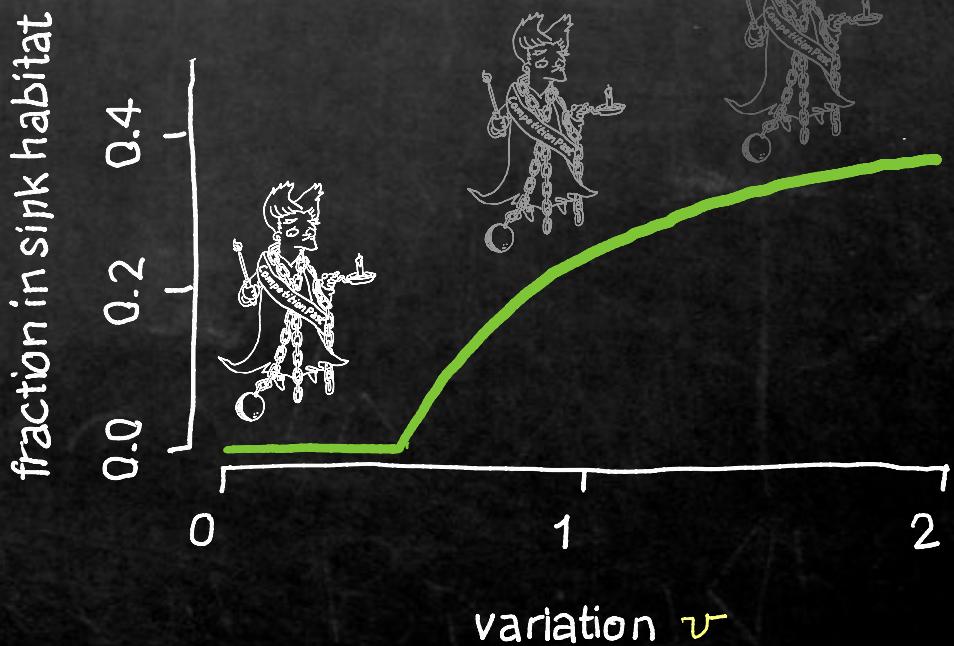
$$r_1 - r_2 = \Delta r \text{ in } k/2 \text{ patches}$$

$$r_2 - r_1 = \Delta r \text{ in } k/2 \text{ patches}$$

spatially uncorrelated  
environ. fluct w/ local  
variance  $v$

$$\frac{v}{2} < \frac{\Delta r}{k} \Rightarrow \text{Ghost of comp. past}$$

$$\frac{v}{2} > \frac{\Delta r}{k} \Rightarrow \text{Exorcism (species co-occur)}$$



## a numerical approach

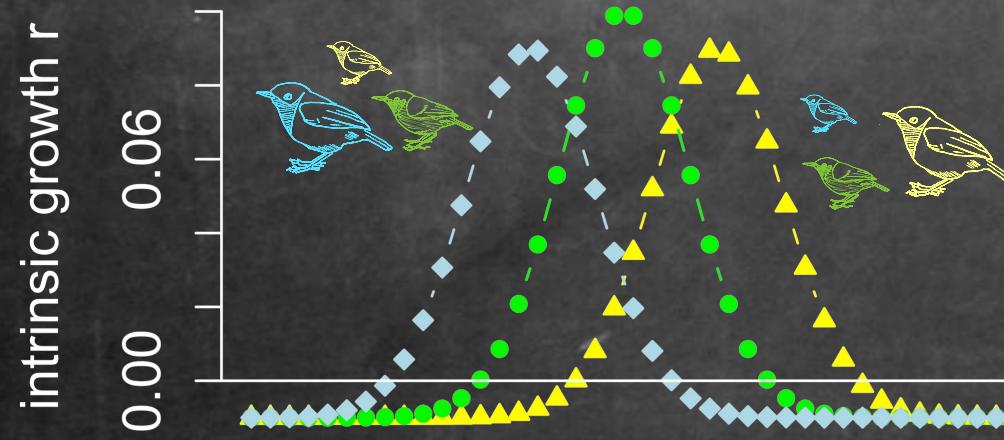
invasion rate

$$d_i(p, q) = \sum_{\ell=1}^k q^\ell f_i(p^\ell \bar{x}^\ell) - \frac{1}{2} \sum_{\ell, m=1}^k q^\ell q^m \sigma_i^{\ell m}$$

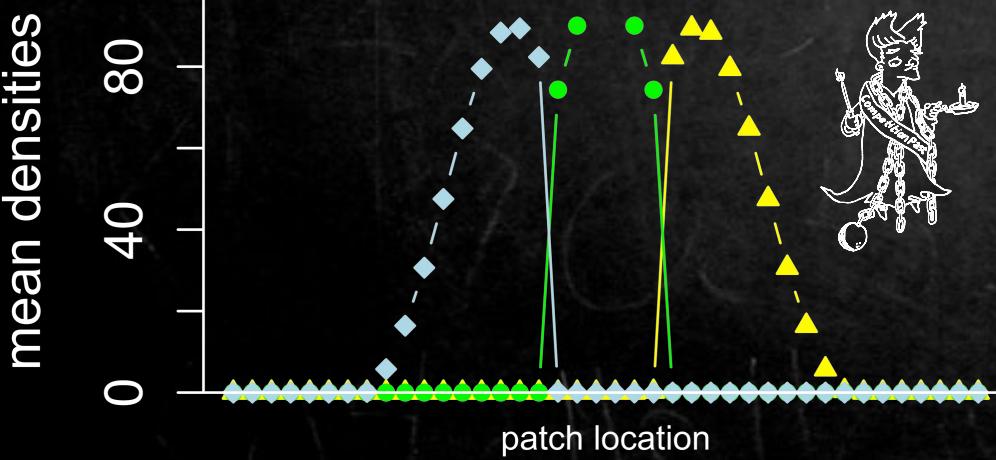
mutation rate  $\mu$

$$dp_i^\ell = \mu p_i^\ell \left\{ \frac{\partial d_i}{\partial q^\ell}(p, p_i) - \sum_m p_i^m \frac{\partial d_i}{\partial q^m}(p, p_i) \right\} dt$$

equilibria satisfy necessary condition  
for coESS

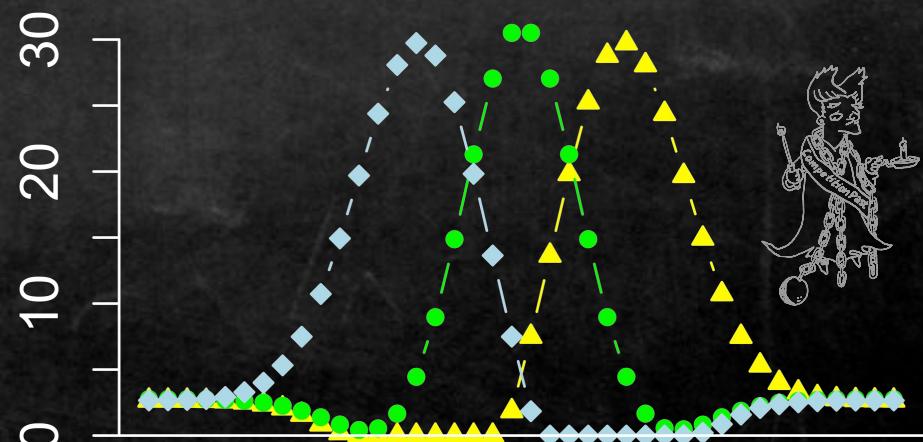


deterministic



patch location

stochastic



2  
sink



## APPLICATION: PREDATOR-PREY

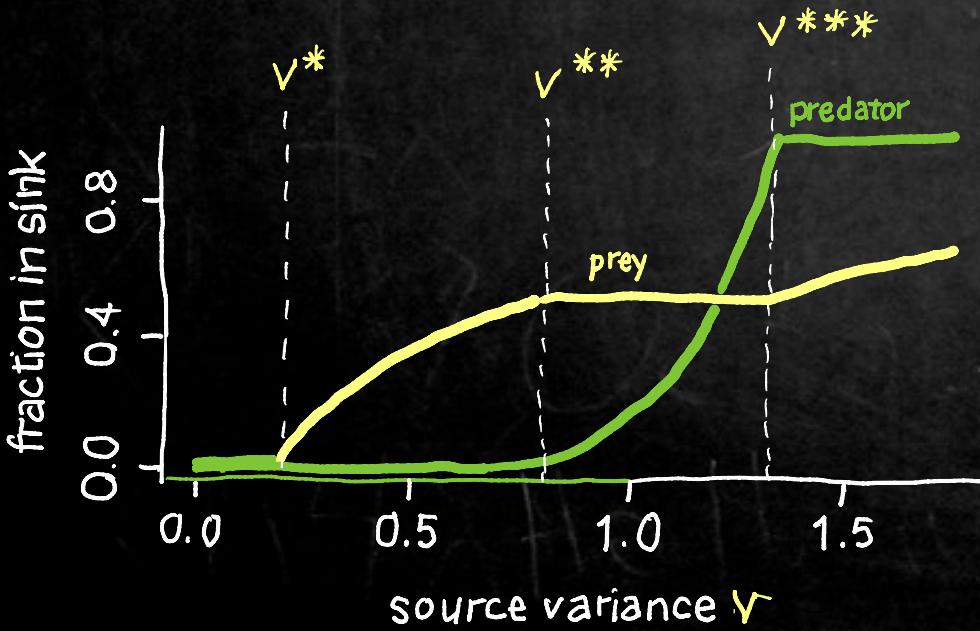
prey

$$dx_1 = x_1 P_1^1 [(r^1 - \epsilon P_1^1 x_1 - \alpha P_2^1 x_2) dt + \sqrt{v} dB] \\ + x_1 P_1^2 [-r^2 - \alpha P_2^2 x_2] dt$$

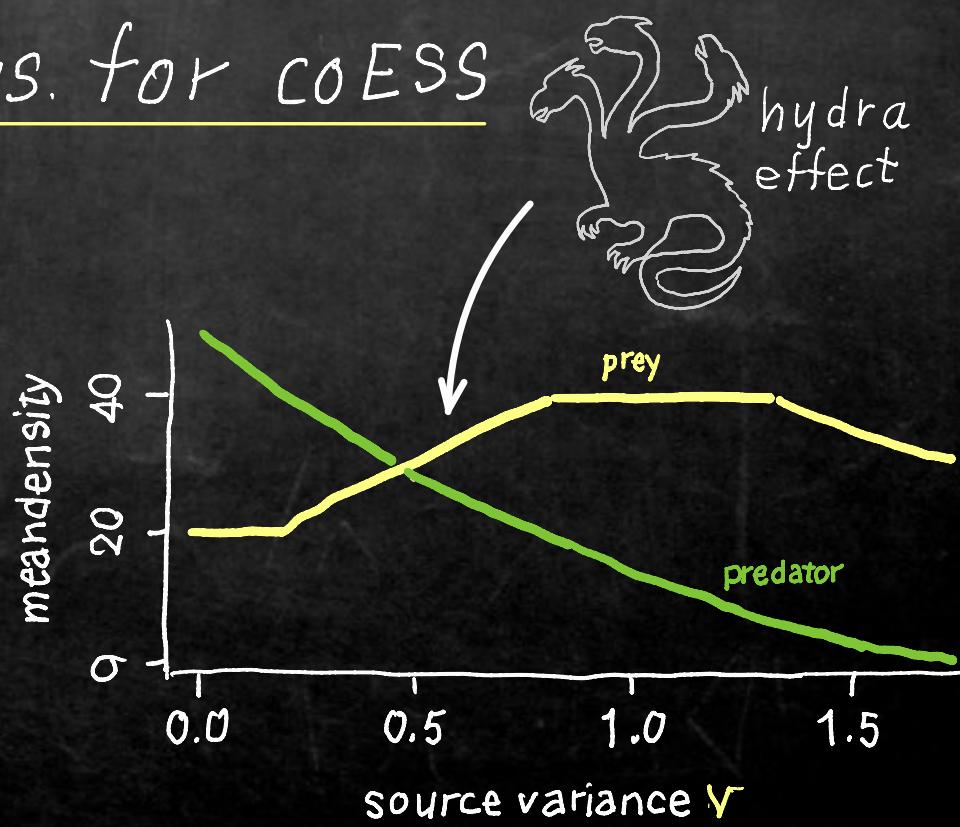
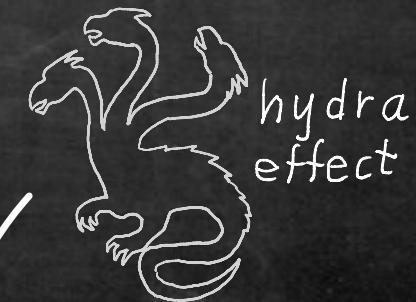
$$dx_2 = x_2 P_2^1 [c \alpha P_1^1 x_1 - d] dt + x_2 P_2^2 [c \alpha P_1^2 x_1 - d] dt$$

|  
predator

source  
1



## Analytic solns. for coESS



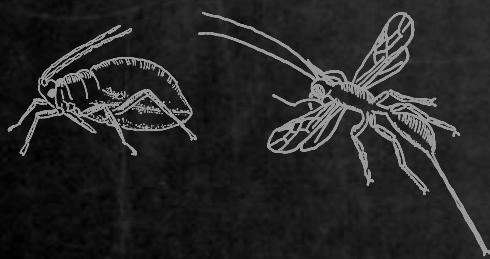
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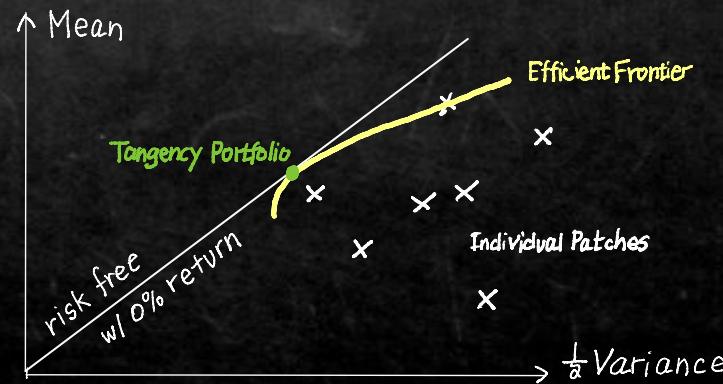


## III. APPLICATIONS: COMPETING SPECIES



## PREDATOR - PREY

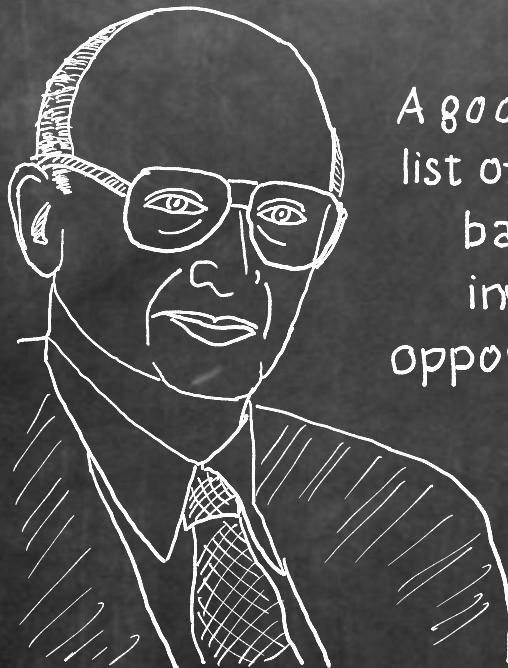
## IV. MODERN PORTFOLIO THEORY & FINALE



# Modern Portfolio Theory: Why It's Still Hip



By BEN MCCLURE | Updated Jan 16, 2020



A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies.

— Harry Markowitz —



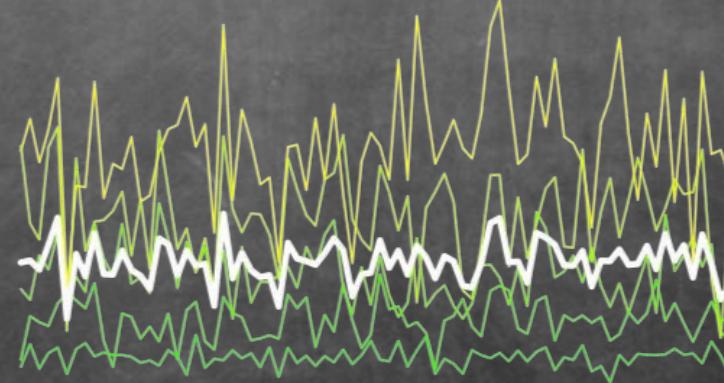
# FINALE

This **KISS** framework extends prior work on evolution of patch selection to metacommunities experiencing spatio-temp. variation

$f_i^\ell - \text{cov}_i^\ell$  equal in all occupied patches and lower elsewhere

temporal var. + species

interactions can select for sink populations, exorcise ghost of comp. past, and select for Hydra effects.



THANK YOU  
FOR LISTENING

&  FOR FUNDING



QUESTIONS?

