# Mathematical Methods in Population Biology

ECL/PBG 231

## Instructor Info


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Pronouns: he, him, his

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Zoom Land

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## Course Info

Prereq: See description

Tues & Thurs

10:30-11.50am

Zoom

## Overview

Course Goals: This course is an introduction to mathematical methods used in theoretical population biology. Population biology, here, refers to any area of the biological sciences focusing on population-level processes including areas in demography, conservation biology, ecology, epidemiology, evolution, and population genetics. The mathematical topics covered will include scalar and multivariate difference/differential equations, matrix and integral projection models, Markov chains, random walks, and stochastic difference equations. As each of these topics, in and of themselves, could take up the entire term, this course is meant as a gentle introduction to each of these topics. Ideally, this introduction will allow you to delve more easily into the theoretical population biology literature, and help you identify which of these topics warrant a deeper dive.

Pre-requisites: In order to focus on the main concepts for this course, I assume that you have a basic familiarity with the following topics (OD refers to relevant pages in the Otto and Day text, see next page):

- Calculus: Definition and interpretation of derivatives and partial derivatives, linearization of single variable and multivariate functions, quadratic approximation of a single variable function (OD 89–106, 303–305)
- Matrix Algebra: basic matrix algebra (i.e. addition, scalar multiplication, matrix multiplication), solving systems of linear equations (OD 214–227, 236–236)
- Statistics/Probability: Definitions of discrete and continuous random variables, expectation, variance, and independence (OD 513–518, 521–526, 536–538).

For those of you wanting a refresher or introduction to these topics, I will provide three optional review sessions on each of these topics during the quarter.

Course Structure, Grades, and Collaborative Learning: To achieve the goals of this course, we will have semiweekly Zoom lectures, semiweekly reading assignments, weekly graded homework, and a take-home final. The lectures, for the most part, will be self contained and the readings are meant to provide an alternative or complementary perspective on the topics. Homework assignments will be a mixture of "pencil and paper" and computer-aided exercises to help you master the mathematical methods and to apply them to different types of models from population biology. The take-home final will be similar to the homework assignments and cover topics from throughout the course.

The grade for the course will be determined by the homework scores (70% of grade) and the take-home final (30% of the grade). Grades will follow the standard scale: A = 90-100%; B = 80-89%; C = 70-79%; D = 60-69%; F <60%.

Everyone is coming into this course from different fields and with different prior knowledge of the material that will be covered in 231. I encourage you to work together as you can teach each other far more than I can. That being said, it is important that homeworks are written up individually, with your own words and calculations, and reflect your own understanding. This will allow me to asses your level of understanding and provide more meaningful feedback.

Diversity and Inclusivity: I consider this virtual classroom to be a place where you will be treated with respect, and I welcome individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, ability - and other visible and non-visible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class.

Sickness/Care responsibilities/Absences: I realize that, especially during this global pandemic, everyone has unique situations and responsibilities which are constantly changing. My primary goal is to support you and your learning and that sometimes requires flexibility. If you have illnesses, care responsibilities, or other scenarios that require you to be absent from class or are affecting your learning, please contact me so that I can work with you to develop a plan.

## Learning Materials

Lectures and Lecture Notes: For this class, lectures and class activities will be held synchronously over Zoom. Please see the syllabus for each section of the course for Zoom links and any additional information. I expect students to join and participate in these synchronous sessions, but recognize there may be circumstances that may make attendance difficult, particularly during the COVID-19 pandemic, so please be proactive about contacting me about these issues as they arise. I will record these Zoom sessions and make them available on Canvas.

To facilitate engagement and discussion, it helps to have videos enabled during class, but I realize there are situations or circumstances that make this difficult or impossible. Thus, I ask you to have your camera on if you can, particularly during discussions. I also encourage you to ask questions during lecture and discussion, which can be done either by using the 'raise hand' icon on Zoom or using the chat function

The most important resource for this course will be your notes from my lectures. Hence, take notes, actively work through the in-class exercises, compare notes with fellow students if something doesn't seem quite right, etc. To help you improve your notes, my lectures will be recorded and posted on Canvas.

Reading Materials: The semiweekly required readings will come primarily from the following two books. For these reading assignments, keep in mind the words of Paul Halmos "Don't just read it" i.e. don't be a passive reader but engage with the reading – how did they do that calculation? why are they making that assumption? how does this idea relate to an earlier idea? etc.

Otto, S. and Day, T. *A Biologist's Guide to Mathematical Modeling in Ecology and Evolution.* Princeton University Press. This book is available as a hardcover (used and new) and as a e-book on Amazon at `https://www.amazon.com/dp/B005N8T8AM/?tag=princetonuniv-20`. You can rent or buy it there from $25.00 (ebook rental) to $61.65 (new hardback).

Edelstein-Keshet, L. *Mathematical Models in Biology.* Society for Industrial and Applied Mathematics. As part of UC Davis, you can download this text for free at `https://epubs.siam.org/doi/book/10.1137/1.9780898719147`

Computational Resources: To explore the concepts visually and with more advanced applications, I will be posting R scripts and links to Java scripts. To use the R scripts, you need to download R at `https://cran.rstudio.com`. In addition, I recommend downloading the interface RStudio Desktop, available freely at `https://rstudio.com/products/rstudio/download/`. If you haven't used RStudio before, watch this $6$ minute video at `https://www.youtube.com/watch?v=5YmcEYTSN7k`. To run the java scripts, you will need to have Java installed on your machine. This is probably already the case, but if not then go to `http://www.java.com`, from where you can both check whether you have Java (and whether your Java is up to date) and download Java.

## Homework Assignments and Final Exam

Homework Assignments: Associated with nearly every lecture, there will be a homework assignment. The assignments will consist of a mixture of "pencil and paper" and computational problems. *The homework problems for any given week need to be turned in on the following Tuesday before lecture through Canvas*. Late assignments will be accepted but with an increasing penalty: $-25\%$ after 10:30am on Tuesday, $-50\%$ after 10:30am on Thursday, $-100\%$ after 5pm on Friday. Assignments need to be submitted as PDFs with solutions to the problems presented in consecutive order. The submissions can be any *legible* mixture of handwritten and hand drawn answers, "typed" (e.g. Word, LaTeX, etc) answers, and images (e.g. plots from R). I encourage you to work together of the homeworks. However, the turned-in write-ups need to be written up individually, with your own words and calculations, and reflect your own understanding.

Take Home Final: During the last week of instruction, I will provide you with a take home final that you need to turn in as PDF on Canvas by 12:30pm on Wednesday, December 16. *No late finals will be accepted.* Unlike the homework assignment, you are not allowed to work together on the take-home final problems and are not allowed to make use of external resources.

## (Tentative) Class Schedule

### MODULE 1: Scalar dynamics

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| Week 1 | Linear difference & differential equations | Reading: OD 191–195,198–202 |
| Week 2 | Nonlinear differential equations I:<br>Phase lines & linearization | Reading: EK 164–171, OD 163–170 |
| | Nonlinear differential equations II:<br>Bifurcations | Reading: Handout |
| Week 3 | Nonlinear difference equations:<br>Periodic fluctuations and chaos | Reading: EK 39–55 |

### MODULE 2: Matrix models

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|---|---|---|
| | Matrix algebra, eigenvalues & eigenvectors | Reading: OD 214–228 (review), 228–230, 233, 237–239 |
| Week 4 | Discrete-time matrix models, I:<br>Non-negative matrices & Perron-Frobenius | Reading: OD 386–397 |
| | Discrete-time matrix models, II:<br>Complex & distinct eigenvalues | Reading: OD 347–361 |
| Week 5 | Continuous-time matrix models | Reading: OD 254–273 |
| | Classification of 2d matrix models<br>& higher dimensional matrix models | Review earlier readings in this module |

### MODULE 3: Multivariate nonlinear dynamics

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| Week 6 | Nullcline analysis and vector fields | Reading: EK 171–181 |
| | Linearization & Phase plane analysis | Reading: EK 181-193 |
| Week 7 | Periodic solutions & chaos | |
| | Invasion analysis | Reading: Handout |
| Week 8 | Discrete-time models | Reading: EK 55–59 |

### MODULE 4: Markov chains and Random walks

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| | Finite state Markov chains, I: Long-term behavior | Reading: OD 608–611, 614–619 |
| Week 9 | Finite state Markov chains, II: Hitting times | Reading: OD 612–614, 621–627 |
| | Thanksgiving | EnJOY |
| Week 10 | Random Walks, I: Law of Large Numbers | Reading: Handout |
| | Random Walks, II: Central Limit Theorem | Reading: Handout |
| Week 11 | the Diffusion equation | Reading: EK 405 (the Box) |
| | Invasion analysis for nonlinear random walks | Reading: Handout |